Handout: Truth, abnormal worlds, and necessity

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MPP conditional

Introduce (or acknowledge) a collection $\mathcal{W}$ of 'worlds' and, in turn, a primitve conditional $\rightarrow$ which is *all-worlds-looking*.

$$w \models A \rightarrow B \text{ iff . } w' \models B \text{ if } w' \models A, \text{ for all } w' \in \mathcal{W}$$

In short: for *any* world w, our new conditonal $A \rightarrow B$ is true at w iff there's *no* world at which A is true but B not.

- Disjunction, Conjunction get expected truth-at-a-world conditions (and falsity- too if need be).
- Negation gets truth-at-a-world conditions that allow for gluts (but, for current purposes, no gaps).
- Validity is as usual: absence of a world that 'makes true' the premises but fails to 'make true' the conclusion.

PMP, Curry

$$\text{PMP.} \ \vdash A \wedge (A \rightarrow B) \rightarrow B$$

Curry paradox combines with PMP and our T-biconditionals to generate triviality (*real* absurdity). Let C be a Curry sentence that says $C \rightarrow \bot$ (e.g., 'If I am true, everything is true'), so that our T-biconditional (dropping $Tr(x)$ for simplicity) gives us $C \leftrightarrow (C \rightarrow \bot)$.

1. $C \leftrightarrow (C \rightarrow \bot)$ [T-biconditional]
2. $C \wedge (C \rightarrow \bot) \rightarrow \bot$ [PMP]
3. $C \wedge C \rightarrow \bot$ [2, substitution]
4. $C \rightarrow \bot$ [3, features of $\wedge$]
5. C [1,4 MPP]
6. $\bot$ [4,5 MPP]

So, we need to avoid PMP!

Abnormal worlds and 'jumpy' conditional

- Our models acknowledge a non-empty set $\mathcal{N} \subseteq \mathcal{W}$ of 'normal worlds'.
- Define all (Boolean or standard first-order) connectives *uniformly* over all worlds.
- For our conditional: acknowledge 'jumpy' behavior, with $A \to B$ behaving one way at normal points and another way at abnormal points.
- For all *normal* worlds $w \in \mathcal{N}$:

$$w \models A \to B \text{ iff . } w' \models B \text{ if } w' \models A\text{, for } \textbf{all } w' \in \mathcal{W}$$

- For all *abnormal* worlds $w \in \mathcal{W} \setminus \mathcal{N}$:

$$w \models A \to B \text{ iff } \ldots \text{[fill in favorite account (say, arbitrary)]}$$

On these 'non-normal-worlds' semantics, we *define validity only over (all)* **normal** *worlds of all models*:

- Validity: no *normal* world (of any model) at which premises true but conclusion untrue.

With this setup, we keep MPP but, as wanted, lose PMP.

- MPP: validity is defined only over normal worlds. At any normal world, $A \to B$ is true iff there's *no* $x \in \mathcal{W}$ at which A but not B is true. Hence, for any normal world w, if $w \models A$ and $w \models A \to B$, then $w \models B$.
- No PMP!! For *abnormal worlds*, we're treating the status of $A \to B$ in an arbitrary fashion. So, just let $\mathcal{W} = \{x, y\}$ with $\mathcal{N} = \{x\}$, and let $y \models A$ and $y \models A \to B$ but $y \not\models B$. Then $x \not\models (A \wedge (A \to B)) \to B$ as there's a point y at which $A \wedge (A \to B)$ is true but B not.

(NB: the Routley–Meyer ternary relation gives a slightly less 'arbitrary' feel to things, but skip this topic here – despite the fact that it is assumed in the background BXTT truth theory in *Spandrels of Truth* (OUP, 2009).)

Recap and Main Issue

- Liars motivates gluts.
- Gluts undermine MMP, and so push for a detachable conditional.
- Worlds and primitive all-worlds-looking conditional gives MPP.
- PMP and Curry paradox require *abnormal worlds* and 'jumpy' conditional.
- ... we have all of this *and* (thanks to Ross Brady) we have a non-triviality proof for truth theories that enjoy such features.

Minimal desiderata for adding Necessity

- Necessitation: If $\vdash A$ then $\vdash \Box A$.
- Box Release (rule): $\Box A \vdash A$.
- Diamond Capture (rule): $A \vdash \Diamond A$ (where $\Diamond A$ is $\neg\Box\neg A$).
- K/Distribution (rule): $\Box(A \to B) \vdash \Box A \to \Box B$.
- S4/KK (rule): $\Box A \vdash \Box\Box A$.

UAW: uniform all-worlds approach

Philosophers usually think of (broad) alethic necessity along 'all worlds' lines. This is a natural start. (The tag 'uniform' concerns no distinction between types of worlds—normal or abnormal.)

- We let $\mathcal{W}$ be our collection of worlds.
- We define our *uniform, all-worlds* (UAW) Box thus:

$$w \models \Box A \text{ iff } w' \models A \text{ for all } w' \in \mathcal{W}$$

UANW: uniform all-*normal*-worlds approach

The current idea is to make explicit use of our (sub-) collection $\mathcal{N} \subseteq \mathcal{W}$ of worlds, namely the *normal* worlds.

- We define our *uniform, all-***normal***-worlds* (UANW) Box thus:

$$w \models \Box A \text{ iff } w' \models A \text{ for all } w' \in \mathcal{N}$$

In short: for *any* world w (of any sort), $\Box A$ is true at w iff A is true at all *normal* worlds (versus, as in UAW, *all* worlds).

Problem with UANW: PMP!

- Define: let $A \Rightarrow B$ be $\Box(A \to B)$.
- Claim: $\vdash A \wedge (A \Rightarrow B) \Rightarrow B$.
- Proof: suppose $w \not\models \Box(A \wedge \Box(A \to B) \to B)$ for some $w \in \mathcal{N}$, in which case there's some $x \in \mathcal{N}$ such that $x \not\models A \wedge \Box(A \to B) \to B$, and so there's some $y \in \mathcal{W}$ such that $y \models A$ and $y \models \Box(A \to B)$ but $y \not\models B$. As $y \models \Box(A \to B)$ we have $z \models A \to B$ for all $z \in \mathcal{N}$, and so *no world* (including y) makes A but not B true. Contradiction.

Diagnosis

- Curry paradox taught that our regular arrow had to be *jumpy*; it had to behave differently at abnormal worlds than at normal ones.
- On our UANW approach, it doesn't matter where in our universe of worlds we are (e.g., a normal or abnormal point); Box claims always look back to normal worlds.
- What's going on, then, is that our UANW approach to $\Box A$ forces A to be evaluated at normal points.
- And that's the problem: PMP is broken only by evaluating parts of it at abnormal points; and $\Box$-ed PMP doesn't get that choice.

JANW: 'jumpy' all-normal-worlds approach

- For all *normal* worlds $w \in \mathcal{N}$:

$$w \models \Box A \text{ iff } w' \models A \text{ for all } w' \in \mathcal{N}$$

- For all *abnormal* worlds $w \in \mathcal{W} \setminus \mathcal{N}$:

$$w \models \Box A \text{ iff } w \models A$$

** Good news: we get the basic desiderata for our necessity operator from this account... [Proof: exercise.]

Actuality: similarly jumpy!

Assuming (as standard) $@ \in \mathcal{N}$, *rigid actuality* must also be jumpy.... [Discuss if time]

$$w \models \alpha A \quad \text{iff} \quad @ \models A$$

Problem: consider $@(A \to B)$!

Overspill result: much more general result!!

In general: there's no sentence that picks out only – or, hence, all and only – normal points! [Discuss if time]

$$w \models \mathsf{n} \quad \text{iff} \quad w \in \mathcal{N}$$

Problem:[1] consider $\mathsf{n} \wedge A \to B$!

[1]Above, n is a proposed sentential – say, 'normal truth' or 'normal-world-here' – constant.